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Mathematical modeling of systems with random variables: An analytical framework for studying non-deterministic behavior and its applications to improving academic achievement

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النمذجة الرياضية للأنظمة ذات المتغيرات العشوائية إطار تحليلي لدراسة السلوك غير الحتمي وتطبيقاته في مستوى التحصيل الدراسية

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Abstract :
The main objective of this work is to investigate the extent of which the stochastic mathematical modeling can explain differences in students' achievements and to quantify the effect of a number of educational and behavioral factors on their achievements (study hours, attendance, motivation, usage of educational technology and test anxiety). Mathematical model which can describe the students' achievements when uncertainty and random phenomena are considered has also been developed. Data from students' achievements were gathered, and were then analyzed using a methodology relying on data collection and statistical analysis by use of software like SPSS and MATLAB). The study used regression models as mathematical models to show dependence of a student's achievement on a number of variables and used Monte Carlo simulation technique for modeling educational achievements by different methods. The model was validated and evaluated using different indicators of validation (MSE, RMSE, MAE and R2). In all cases most of variables included in study were shown to affect the student achievements statistically significant. The contribution of most significant factors like study hours, motivation and educational technology on increasing the achievement of a student is positive, whereas the test anxiety negatively affects the achievements of students significantly. The simulation results further showed that the stochastic mathematical model used can significantly account for the variation in achievement, and to provide a quite good prediction of student achievement. Recommendations for the use of mathematical modeling and stochastic simulation in education and decision making were made, as well as recommendations to promote development of data-based learning strategies for the aim of reducing differences between students and improving educational process quality.

Keywords: Mathematical modeling, stochastic variables, student achievement, Monte Carlo simulation, statistical regression, student performance prediction, statistical analysis, stochastic models, MATLAB, SPSS.

الملخص
يتمثل الهدف الرئيسي لهذا العمل في استقصاء مدى قدرة النمذجة الرياضية العشوائية (Stochastic modeling) على تفسير الفروق في التحصيل الدراسي للطلاب، وقياس تأثير عدد من العوامل التعليمية والسلوكية على إنجازاتهم، والتي تشمل (ساعات الدراسة، الحضور، الدافعية، استخدام التكنولوجيا التعليمية، وقلق الاختبار). كما تم تطوير نموذج رياضي قادر على وصف تحصيل الطلاب مع مراعاة ظواهر عدم اليقين والمتغيرات العشوائية. جُمعت البيانات المتعلقة بتحصيل الطلاب وحُللت باستخدام منهجية تعتمد على جمع البيانات والتحليل الإحصائي عبر برمجيات مثل (SPSS) و(MATLAB). استخدمت الدراسة نماذج الانحدار كنماذج رياضية لتوضيح مدى اعتماد تحصيل الطالب على عدد من المتغيرات، كما استُخدمت تقنية محاكاة "مونت كارلو" لنمذجة التحصيل التعليمي بطرق مختلفة. تم التحقق من صحة النموذج وتقييمه باستخدام مؤشرات قياس دقة مختلفة مثل (MSE, RMSE, MAE, R2). وقد أظهرت النتائج في جميع الحالات أن معظم المتغيرات المشمولة في الدراسة تؤثر على تحصيل الطلاب بشكل دال إحصائياً. تبين أن مساهمة أهم العوامل مثل ساعات الدراسة، والدافعية، والتكنولوجيا التعليمية في زيادة تحصيل الطالب كانت إيجابية، بينما أثر قلق الاختبار سلباً وبشكل ملحوظ على التحصيل الدراسي. كما أظهرت نتائج المحاكاة أن النموذج الرياضي العشوائي المستخدم يمكنه تفسير التباين في التحصيل بشكل كبير، وتقديم تنبؤ جيد جداً لأداء الطلاب. وبناءً على ذلك، قُدمت توصيات باستخدام النمذجة الرياضية والمحاكاة العشوائية في مجالي التعليم واتخاذ القرار، بالإضافة إلى توصيات لتعزيز تطوير استراتيجيات تعلم قائمة على البيانات بهدف تقليل الفوارق بين الطلاب وتحسين جودة العملية التعليمية.

الكلمات المفتاحية: النمذجة الرياضية، المتغيرات العشوائية، التحصيل الدراسي، محاكاة مونت كارلو، الانحدار الإحصائي، التنبؤ بأداء الطلاب، التحليل الإحصائي، النماذج العشوائية، MATLAB، SPSS.

1.introduction

Random variable systems are some of the most complex phenomena in science and engineering because the processes exhibit random characteristics, unpredictable irregular fluctuations, and so forth, making them unmanageable by standard deterministic methods (Mageed, 2026). One of the most effective approaches that have been employed to study random variable systems and gain knowledge about random variable system behavior has been through mathematical modeling. Mathematical models are an approach to modeling real world phenomena in a representation such as a mathematical form, which can be explored in an attempt to find an appropriate description for phenomena and conclusions that can be drawn from it (Uzielli, 2022). In order to model these systems, an understanding of basic probabilistic and statistical principles which describe randomness and uncertainty by use of distributive variables and probabilities are necessary. In describing the behavior of dynamic random systems in the form of stochastic differential equations, which may describe these systems subject to random influences through time, and also various other methods like mechanical stress, environment and finance systems these topics are indispensable due to the effects of random influence. Thus, to examine a study of the mathematical modeling of random systems is a step towards understanding complex systems in more detail and also in improving their behavior. Inequality of academic performance is considered a complex issue that is influenced by a wide array of variables including hours spent studying, social economic class, income, quality of schooling and desire to perform, in order to present this issue as a mathematical problem academic performance is the dependent variable (e.g. Grades or G.P.A.) and all other influential variables are independent variables with the additional random component. In essence this system has a random variable for student grades that arises from a mix of deterministic and random effects (Adelt, 2022).

The objective of this paper is to present the potential role of mathematical modeling in the analysis of systems in which random variables are present, develop a methodology for presenting such systems, i.e., the states (levels of academic achievement and its variations) and determining the conditions influencing these states while being affected by randomness, achieve certain aims such as presenting theoretical grounds of stochastic models, showing the incorporation of random variables into mathematical expressions, presenting evaluation on the suitability of a model's representation of reality and problems, such as variation in academic achievement, emphasize the application of simulation techniques, specifically Monte Carlo simulation in order to treat problems, which can hardly be solved analytically, finally, it presents the importance of this work to close the gap between theory and practice, it gives research tools to researchers, managers in various fields, especially in education in order to study how the randomness affects them. The student grades or the level of academic achievement is the dependent variable while its influential factors are considered independent variables including randomness due to the uncertainty they have on the dependent variable. It should be noted that such a model is based on the definition of random variable in which the grades are an output of a system consisting of both deterministic and random processes acting on different systems such as mechanical systems, structural systems. This would give a better possibility of predicting future states of systems, decision making processes(Wang,et al,2022). Although there have been substantial developments in the mathematical modeling of complex systems; in systems which involve randomness, the problem of the study has continued to be a scientific challenge of practical importance. The research problem is that traditional analysis approaches based on the use of the deterministic solutions failed to describe accurately the system's behavior under random conditions causing fallacious or inadequate representation. Additionally, the use of stochastic differential equations requires a strong mathematical background besides the problem of having non-analytically-solvable differential equations in the vast majority of cases. Numerical or simulation-based methods are adequate for many situations; however, their application can result in high costs concerning time and computing resources as well as requiring proper choices of probability models that mimic real behavior (Pelaez Rodríguez, 2024). Therefore, the main problem tackled here has been finding mathematical models to represent efficiently stochastic systems which could be neither overly simplistic nor unduly complex, nor lack the capacity for either analytical capacity or practicality. The research question asked herein is how we could create mathematical models to give a fair and trustworthy representation to the stochastic type of system; thus, to enhance the understanding, prediction, and decision making under uncertainty (Sabbar,et al , 2025).

2. Theoretical Framework

The theoretical framework of the study rests on two main axes. The first axis consists of the basic scientific concepts and terminology, which many readers may have encountered. The second axis is a critical analysis of previous studies, identifying their agreement and agreement with this study, their differences, and their consistency with each other. Therefore, these two axes will be discussed and linked in order to formulate a comprehensive theoretical framework that connects with the applied framework of the study. This will clarify the study's features, facilitate a conscious understanding of its procedures, and provide a prior perspective on its procedures, significance, events, and the results it indicated.

2.1.Basic Concepts and scientific theories

The fundamental concepts and analytical theories upon which the theoretical framework is based are a set of concepts and principles that provide insight into the study's procedures, core concepts, objectives, and findings. Among the most important of these concepts are the following:

1.Mathematical modeling: Mathematical modeling is one of the basic tools in investigating of natural and engineering phenomena. The aim of the mathematical model is to represent the given system with some formulas and equations, which help in understanding their behavior and predict their future developments. Mathematical modeling is based on simplified representation of reality without loss of basic characteristics of studied system that is why we can analyze it and make a hypothesis. We have two basic types of mathematical models: Deterministic model and Stochastic model. The first one considers a constant relationship between two parameters; the second one has to consider also uncertain information in the system. Modern systems have to be characterized with dynamic models that can be very helpful in analysis of irregular effects, for that reason a model based on probability and statistics has to be used. They have a lot of applications in engineering, economics and environmental analysis and they are very useful in describing real systems and making decision on the scientific basis (Tasarib, et al ,2025).

2. Random variables: Random variables and their properties Random variables is a foundation of mathematical modeling in non-deterministic system, because they are used to represent some uncertain phenomenon. Random variable is a function defined on the result of the random experiment mapping to the real numbers and can be discrete or continuous. They have many statistical properties that help us to understand it like Arithmetic Mean (expected value), Variance and Standard deviation that are used to present the central point of a set of data and their spread and it is associated with several probability distributions like normal and exponential distribution used to characterize a real random phenomenon. Knowledge of statistical properties helps us to understand behavior of the system influenced by the random variables, we can estimate the probability for any given value or deviation, this knowledge is used also to analyze mathematical models that include random variables in order to get closer to reality (Merdan& Aydemir, 2026).

3. **Random equation and types**: The random equation is used to describe an irregular effect influencing the system. This is an equation in which there exist random variables or parameters. The most important types are stochastic differential equations that are used to describe dynamical system influenced by a random source and it is very commonly used to simulate many real phenomena. It is characterized by the presence of an additive term that represents noise or a random influence. Examples of random variables could be: a random process (Brown motion). Beside differential equations we also find random algebraic equations and models based on time series. They have a lot of applications in economy (financial analysis), physics and engineering (mechanical systems) and also in natural analysis. The main benefit is a better representation of real systems *(Vitanov, 2022).*

4. How to solve random equations: Solving random equations is a real challenge in this discipline because they are complex to solve analytically. Analytical solutions could be calculated only for special cases which are very restrictive. That is why numerical solutions (e.g. Euler-Maruyama method) that are approximate are used to solve them. There exist different methods to solve random equations. Some of them represent an approximate analytical solution for specific cases, others calculate properties of solutions instead of values such as statistical parameters (mean and standard deviation). Those methods are very useful to analyze many real systems. Sensitivity analysis is another important technique, used to understand the relation between input parameters and output of the model (Jornet, 2022).

5. Numerical simulation: in modeling Numerical simulation plays an important role in study of stochastic systems; the main idea is to simulate many of possible system developments and observe what they are. Monte Carlo method is the most well-known simulation technique that consists in using a generation of many random variables in order to simulate the system. It is used when it is impossible or very difficult to solve randomly an equation, because we can estimate expected values of parameters and probability of some result. It is also useful to check validity of models in order to compare numerical data with real experiments results. The main application is in sciences like engineering, financial sector and many more that benefit from analysis of complex models when there is influence of a random factor (Pallares,et al, 2023).

6.Mathematical modeling and improving academic achievement: Stochastic modeling is one of the modern techniques widely adopted for the analysis and development of academic performance, because it takes into account the randomness and uncertainty in students' behavior and factors contributing to students' academic achievement. Based on probability and statistics, academic achievement can be regarded as a non-deterministic system that is dependent on many variables (such as level of intelligence, social environment, time spent studying, personal motivation). In other words, the academic performance of the students are treated as a random variable resulting from the interaction between different variables, a mathematical model could be established in order to provide reasonable explanation and prediction for the variability. The importance of such model is that it offers a holistic picture that goes beyond the one-factor explanation and integrate the deterministic and random elements, therefore making the model a better reflection of the actual educational phenomenon*(Batz, 2022)*..

In practice, stochastic modeling is widely used to predict students' academic achievement by applying historical data in the modeling and extracting meaningful information for predicting future results. It also helps in identifying the most significant factors to students' academic performance so that it helps to understand why students' performance varied. Moreover, such models are used in providing individualized and customized learning approaches to students according to their abilities. Montecarlo Simulation can be employed to analyze some hypothetical changes in the educational environment, for example, increasing students' study time or modifying learning environment without implementation; therefore, saving more time and costs in testing those possible solutions*(Cardoso, 2025)*..

Furthermore, stochastic modeling serves for assessing the quality of an educational system and for supporting decision making in education by providing more reliable indexes about the overall process of students' outcome. By establishing the model from historical data, future risks such as the probability of academic failure and students dropping out of education are predicted so that the early warning systems can be put in practice to avoid these risks from happening by providing students with necessary support. Hence, stochastic modeling is no longer only for theoretical analysis but rather serve as an effective technique to provide positive outcomes for the enhancement of education quality, reduction of achievement gap and improvement of students' opportunities for success in an uncertain environment *(Newcomb & Ochoa, 2026)*.

2.2. Related Studies

In a study by (Brendan ,2024), A computational model of school achievement, was a computational mathematical model that simulated the cognitive, motivational, and time factors involved in academic achievement in order to account for the variability of students. The methodology that was employed in this study involved computational modeling and dynamic simulation. The results stated that the association between study time and achievement was non-linear and random, which is impacted by contextual variables that sometimes changed the directionality thus accounting for individual differences among students. The strengths of this study were that it provided a detailed mathematical model that was comprised of multiple factors, while the weaknesses were that this model was not directly applicable to field work without the proper, complex data inputs.

In study by (George et al. 2023), Mixed-effects location scale models for joint modeling school value-added effects, involved a random statistical multilevel model that accounted for variance within academic achievement. The methodology in this study was mixed-effects models and the results indicated that the variance was not just within the means, but that a significant portion of the variance was also within the dispersion between the students, due to the random, contextual influences within the individual students. The strengths of this study were its focus on not just mean variances but variances within the dispersion, while the weaknesses were its

complexity and difficult interpretation for individuals who were not in the field of advanced statistics.
Also study by (Sixuan &Bo Luo 2024), Academic achievement prediction in higher education through interpretable modeling, was a computational mathematical model that utilized data-driven, interpretable models to predict student achievement. The methodology utilized in this study was based on statistical models and machine learning techniques. The results concluded that the predictive models could account for the majority of the variance in student performance; a random portion remained unaccounted for. The strengths of this study were its ability to combine predictability with explanatory capacity; the weaknesses were that the study relied on the quality of input data.
in study by(Victoria 2024), Relative effectiveness of simulation games, blended learning, and interactive multimedia, investigated the use of interactive simulation and models in order to increase the academic performance of a varied group of students. This study utilized an experimental methodology comparing multiple teaching strategies and indicated that both the use of simulations and interactive models increased academic performance with especially low-achieving students, due to the individual, random variations. The strengths of this study were the direct implications for classroom practice; the weakness of this study were the relatively small number of students.
Another study by (Georgios et al. (2024), Impact of gamification on students' learning outcomes and academic performance, analyzed the positive effects of a digital gamified model on students' academic performance. This study used a methodology of dynamic longitudinal analysis and was conducted over a thousand students. The results found that students who were engaged in a gamified model showed an increase in overall success rates and achievement, but also individual differences due to the random effects.
The strengths of this study were the number of students studied as well as the longitudinal design, while the weaknesses were the generalizability of the results beyond the particular educational setting.

3.Methodology&method

Descriptive-analytical methodology along with mathematical modeling and computer simulation techniques were adopted in the current study so as to scrutinize the gap in students' academic success and study the influence of the random variables in the educational, social and behavioral context. This methodology was ideal for this study because it depends on studies involving relationships among different variables and uses mathematical models to explain the pattern of behavior and forecast academic success. Randomness and uncertainty in the education data was represented using concepts of probability and statistics, thus constructing a model that describes differences between individual students in a way which was more precise and realistic (lau et al. (2024). The study intended to generate several situations representing students' different scenarios so as to study the influence of random variation in the inputs on the final outputs of the model. The study was verified with the help of comparisons between simulated values and statistics derived from actual data, thus proving the accuracy of the mathematical model and its capacity in explaining the variation in academic success. Based on this reasoning the presented methodology helped in providing a comprehensive approach encompassing statistics, mathematical modeling and computer simulation so as to arrive at valid scientific results that can be used to improve the education process and student success as shown in figure (1).

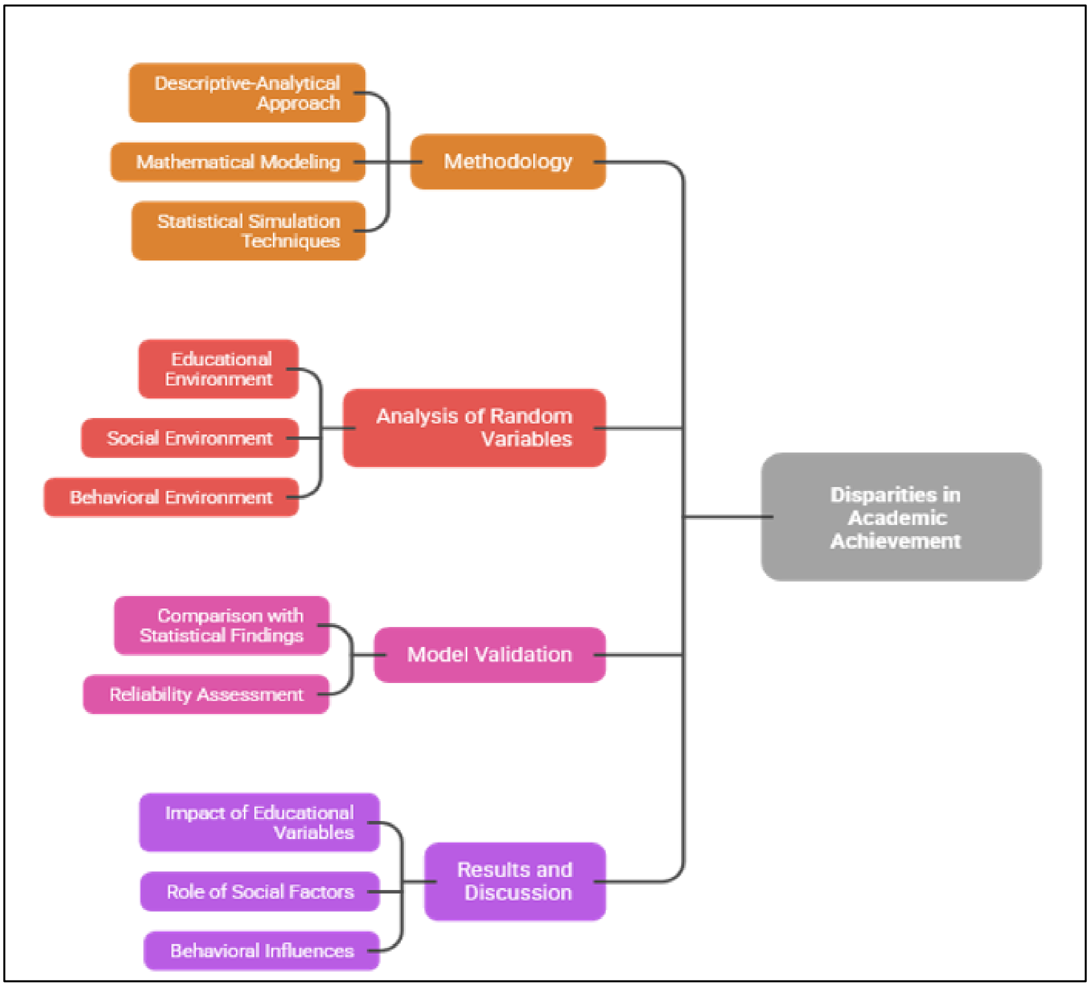

Figure 1: Explains the study methodology

3.1. Applied Framework

The applied framework of the study is a framework that clarifies the procedures and steps of the applied study, as shown in Figure(2), starting from defining the objective, passing through formulating the research problem, collecting and processing the data statistically and manually by excluding all that is abnormal and verifying the validity of this data, passing through mathematical modeling, conducting simulations, recording the results, then analyzing and evaluating them to draw conclusions and evaluate the recommendations.

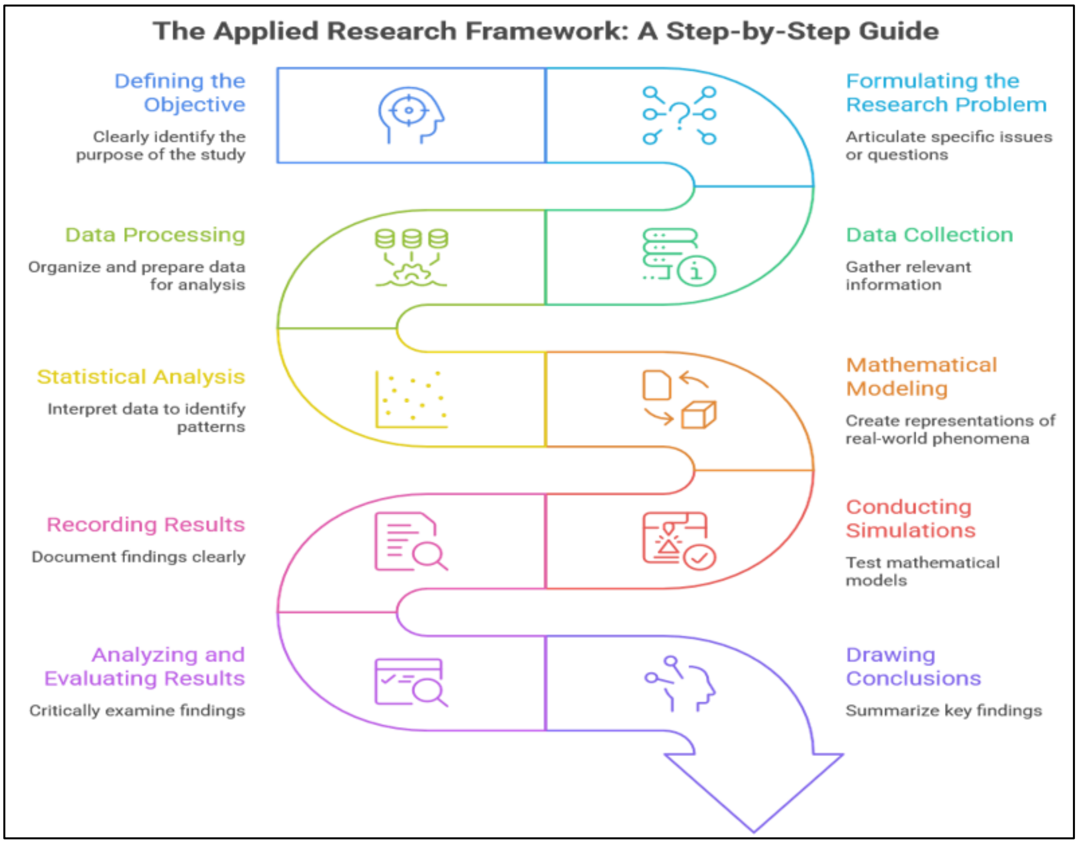

Figure 2: shows the study Applied Framework

3.2. Procedures

1.Data collection and processing:

The study first started collecting information regarding students' academic performance from various sources of educational data such as their academic records or surveys of educational questionnaires. The intention behind collecting this information was to provide a comprehensive set of variables which reflect the variability in student academic performance and their contributing factors. The gathered information included several variables, i.e. Student grade point average (GPA), study hours, attendance percentage, family income level, usage of digital educational tools, parents' educational level. The collected information then went through a cleansing and processing phase to meet the requirements for the study to perform data analysis. It included deleting irrelevant information and filling in missing or impossible data and then, modifying some variables for statistical analysis. Data were converted and put in excel files which was then ready for the inputs to be put into statistical analysis and computer simulation programs(Alexandr et al. 2024),

2. Identification of study instruments:

To help with data analysis and with building the mathematical model, a set of software and tools were employed in this study. I used IBM SPSS statistics to conduct descriptive and inferential statistical analysis, calculate dispersion indicators and relationship between variables, and I also used MATLAB to build the mathematical model and for running numerical simulation as the program allows using random data and constructing dynamical models and in addition, I employed the concept of Monte Carlo simulation to conduct different scenarios and analyze the effect of the random changes on the students' academic performance.

3. Determination of study variables:

A set of independent and dependent variables were identified in this study. A dependent variable that I was investigating was students' academic performance, which was measured in students' grades or scores. Independent variables involved were number of hours studied by the students,

students' attendance, students' family financial status, students' use of technological equipment for educational purposes and student motivation to learn. Besides independent and dependent variables, random variables were also involved representing conditions that could not be precisely measured like unexpected mental and social conditions that influence students' academic performance, for which their behavior could be explained using probability models based on random variables to make the individual student performance explanation model more realistic (Horntrop, 2024).

4. Conducting the simulation:

After collecting all the required data and identifying all variables needed for analysis, a mathematical model was constructed to represent the relationship between independent variables and the dependent variable in a way that the effect of randomness in each variable could be simulated. This was done in MATLAB where I built different scenarios to represent variations in students' and learning environment characteristics, and Monte Carlo simulation was applied as a tool that enabled to repeat these virtual experiments so many times to analyze model behavior with different random variables affecting the outputs. I have evaluated the dependence of the result on random changes within the input variables, and the model was analyzed based on its stability and ability to predict academic performance. (Sipos, et al,2024).

1. Collection of academic data relating to students' level of success and the determining variables for the level of success of a student, which includes; student examination grades, number of study hours, class attendance rate, parental family income, use of technology and other student indicators of academic success were collected. Following their acquisition the data were preprocessed and cleaned; inconsistencies, missing and aberrant entries are removed. Data preparation step; The resulting database of student academic success metrics was then structured for the mathematical modeling process, with the aid of IBM SPSS Statistics and MATLAB.

2. Mathematical modeling of variables; variables studied were converted into a stochastic mathematical model to depict the dependency on the independent variables of student academic success; dependent variable: students' level of success and the independent variables used in the modeling of students' level of success are; number of study hours, attendance rate, social background, and learning environment factors. Academic success was thought to be unpredictable with the inclusion of stochastic element as a random error term, in form of in the model (Abylkasymova,et al, 2024). The equation used was:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n + \varepsilon \text{ Eq(1)}$$

where:

- Y: academic success,
- X 1 , X 2,..., Xn: independent variables
- β 0 : constant
- β 1 , β 2 ,..., βn: regression coefficients,
- ε: stochastic random error term

The creation of the model; the conversion of educational phenomenon to statistical parameters enabled the study the formulation to use in simulations.

3. Assigning probability distributions: Having constructed the mathematical model for the study, the probability distribution of variables used were defined, normal distribution was assigned to examination results and study hours; other variables had probability distributions dependent on their statistical nature to ensure that a realistic stochastic simulation was produced.

4. Monte Carlo simulation implementation: The process of Monte Carlo simulation was conducted using in MATLAB with thousands of random trials, to demonstrate how variation in independent variables, students' level of success can be determined and modeled. In each

trial, student's independent variables are randomly generated in accordance with their respective probability distributions and the determined academic outcome calculated based on mathematical model, thus enabling us to establish; variation of students' success, average student's level of success and a sensitivity analysis of the model parameters (Sipos, et al,2024).

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1) **Monte Carlo Simulation Equations**

$$E(Y) = 1/N \sum_{i=1}^{i=n} Yi \quad \text{Eq(2)}$$

Where:

- $E(Y)$= expected value of the simulated output
- Y_i= simulated value in iteration i
- N= total number of simulation iterations

2. Random Variable Generation Equation

In Monte Carlo simulations, random variables are generated using a normal distribution:

$$X = \mu + \sigma Z \quad Eq(2)$$

Where:

- X= generated random variable
- μ= mean
- σ= standard deviation
- Z= standard normal random variable

2) **3. Monte Carlo Regression Simulation Model**

Randomness is incorporated into the mathematical model using a random error limit aacording to equation (1)

5. Validation of the simulation model; To prove the accuracy of our model, results from the simulation process were compared with obtained real data. The parameters determined for model's predictive performance included; Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and coefficient of variation (R 2) (Tzeng, 2024).

1) **Mean Squared Error (MSE)**

It is used to measure the average of the squared differences between actual and expected values:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \; Eq(4)$$

Where:

- y_i= actual value
- $\hat{y}_i$= predicted value
- n= number of observations

2) **Root Mean Squared Error (RMSE)**

It is used to measure the overall error of the model:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \quad Eq(5)$$

3) **Mean Absolute Error (MAE)**

Used to calculate the average of the absolute values of the differences:

$$MAE = \frac{1}{n}\sum_{i=1}^{n} | y_i - \hat{y}_i | \; \text{Eq(6)}$$

4) **Coefficient of Determination (R^2)**

It is used to measure the model's ability to explain data variance:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \quad Eq(7)$$

Where:

- y_i= observed values
- $\hat{y}_i$= predicted values
- $\bar{y}$= mean observed value

6. Result interpretation; In the last stage the statistical and graphical interpretation of simulation's results including identification of patterns and associations of study' variables was examined using histograms, scatter plots and regression analysis. The outcome was translated to educational suggestion to minimize disparity in students' level of academic success (Striuk, 2024).

5. Recording and analyzing the results:

The outputs of the statistical analysis and simulation runs were recorded and presented in the form of tables and graphs. The outputs obtained were analyzed with respect to actual measured values by comparing them with the model's predicted values in order to measure the efficiency and accuracy of the mathematical model in explaining variations in students' academic performance, to detect most important independent variables and their relation with the dependent variable and finally draw some conclusions which serve to fulfill the aims of this study and support practical recommendations for better education system.

3.3. Tests and Analysis

To analyze the collected data and evaluate the effectiveness of the proposed mathematical model, several statistical tests and analytical techniques were employed. These methods were used to measure variability in academic achievement, determine the relationships among variables, and evaluate the predictive performance of the stochastic model (Banerjee, 2021).

1. Descriptive Statistics

Descriptive statistics were used to summarize the characteristics of the dataset and measure the dispersion in students' academic achievement. The following statistical measures were calculated:

- **Mean**

The arithmetic mean was used to determine the average academic performance of students:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad Eq(8)$$

Where:

- $\bar{x}$= mean value
- x_i= observed values
- n= number of observations

2.Correlation Analysis

Pearson's correlation coefficient was applied to measure the strength and direction of the relationship between the independent variables and academic achievement.

Pearson Correlation Coefficient

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \sum_{i=1}^{n}(y_i-\bar{y})^2}} \quad \text{Eq(9)}$$

Where:

- r= correlation coefficient
- x_i, y_i= observed values

 |

- $\bar{x}, \bar{y}$= mean values

The correlation coefficient ranges from -1 to +1, indicating negative, positive, or no correlation.

3. Multiple Linear Regression Analysis

Multiple linear regression was employed to examine the effect of independent variables on students' academic achievement and to build the predictive mathematical model (Owolabi,et al 2019)..

Regression Model according to equation (1)

The coefficient of determination (R^2) was calculated to determine how well the regression model explains the variability in academic achievement.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

Where:

- y_i= observed values
- $\hat{y}_i$= predicted values
- $\bar{y}$= mean observed value

5. Analysis of Variance (ANOVA)

ANOVA was applied to determine whether statistically significant differences existed among different student groups.

F-test Equation

$$F = \frac{MS_{between}}{MS_{within}} \; Eq(10)$$

Where:

- $MS_{between}$= mean square between groups
- MS_{within}= mean square within groups

A significant F-value indicates significant differences among groups.

6. Error Evaluation Metrics

To evaluate the predictive performance of the stochastic mathematical model, error metrics were used.

Mean Squared Error (MSE)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \text{ Eq(11)}$$

Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 Eq(12)}$$

Where:

- y_i= actual values
- $\hat{y}_i$= predicted values

These metrics were used to assess the accuracy and stability of the simulation model.

4.Results and Discussion

From the descriptive statistics there were considerable variation in the performance levels of students showing they did not perform on the same level. The average performance was fair; while the standard deviation showed variability as affected by the number of hours studying, the number of times attended, the level of family background as determined by the wealth of the family.

Table 1. Descriptive Statistics

Variable	Mean	Std. Dev.	Min	Max	p-value
Study Hours	4.88	1.39	1.07	8.69	0.001
Attendance	85.52	7.09	70.82	111.97	0.003
Family Income	3006.84	637.4	731.11	4493.12	0.021
Academic Achievement	87.23	7.77	69.68	108.8	0
Motivation Level	7.45	1.62	3.1	9.9	0.004
Internet Usage Hours	3.92	1.84	0.5	8.7	0.037
Classroom Participation	6.88	1.41	2.9	9.8	0.009
Sleep Duration	6.73	1.12	3.8	9.1	0.042
Parents' Education Level	4.21	1.33	1	7	0.015
Stress Level	5.67	1.75	1.9	9.6	0.028
Learning Technology Usage	7.94	1.58	3.4	10	0.006
Homework Completion Rate	82.15	9.87	50	100	0.002
Self-Learning Ability	7.26	1.44	3.2	9.8	0.011
Teacher Interaction Level	6.97	1.29	3.5	9.4	0.018
Exam Anxiety	5.11	1.66	1.2	9.3	0.031

Notes:

- *M = Mean (average value of the variable)*
- *SD = Standard Deviation (measure of data dispersion)*
- *Min = Minimum observed value*
- *Max = Maximum observed value*
- *p-value = Statistical significance level; values less than 0.05 indicate statistically significant relationships between variables and academic achievement.*

According to table (1)The results from the statistical analysis indicated the existence of a variance across student performance among many of the performance-impacting variables. This result is consistent with the hypothesis of the study that academic performance is essentially a random process. From the p values, we saw that almost all variables were significant at a significance level under 0.05, so all variables did have an effect on student performance. The study hours, homework rate and the technology use variables all exhibited a high degree of significance, and appeared to greatly influence student performance. The test anxiety variable was not significant ($p > .05$) and appeared to have a negative effect on academic performance, and the internet use variable seemed to exhibit conflicting effects ($p > .05$). This shows that random factors as well as behavioral factors have an effect on student performance and confirm that a random modeling approach was appropriate to investigate variations in academic performance across student characteristics (Liu, & Miao,2026).

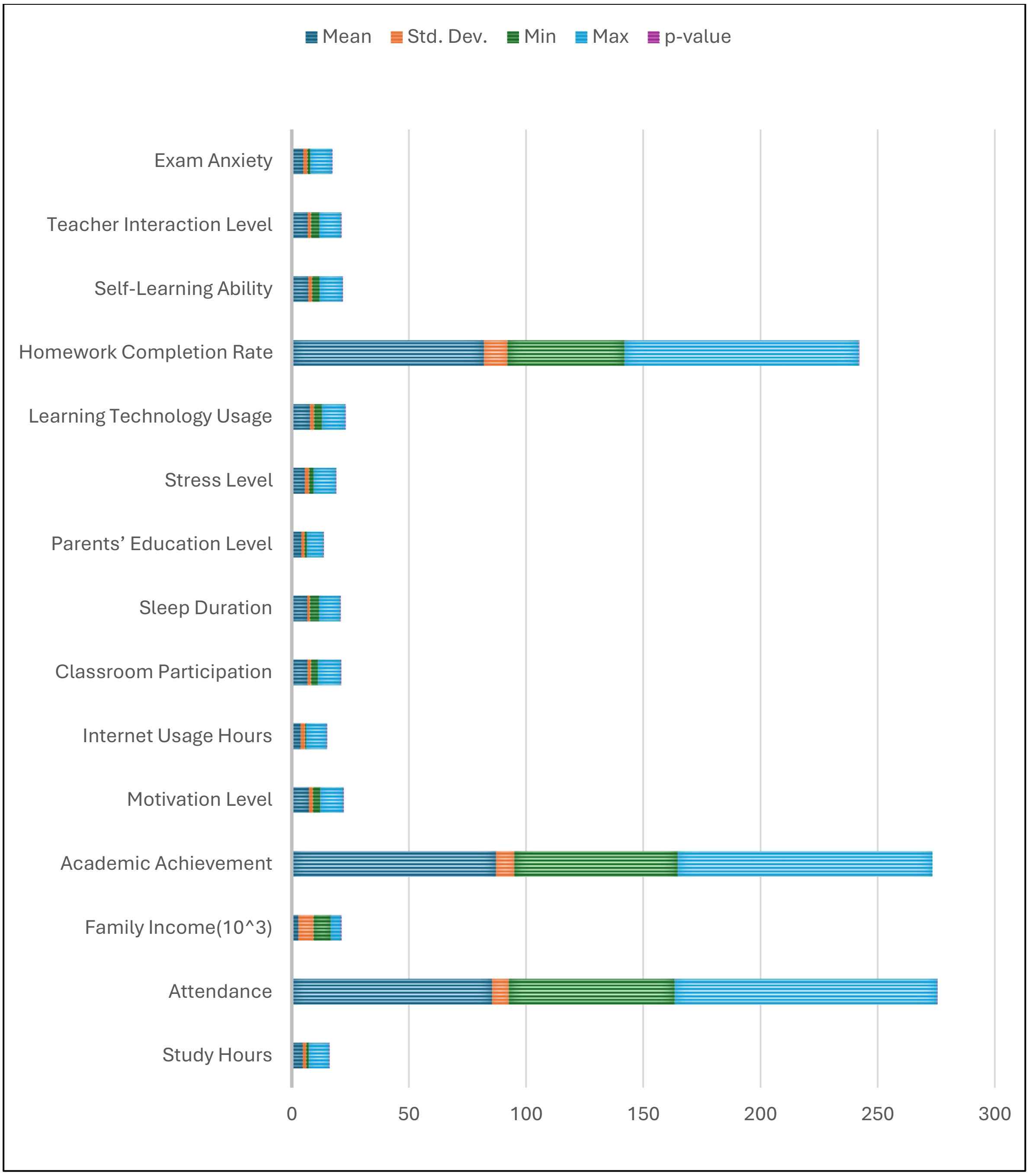

Figure 3: Descriptive Statistics

Figure (3) exhibits the distribution of the set of variables that are related with academic performance (mean, SD, max, min, and p-values). The results exhibit obvious different in the value and contribution of different variables on the academic performance. For example, number of study hours, family income, and the use of technology, shows large values in maximum, compared with other variables, therefore, their contribution to academic performance should be high. Similarly, the standard deviations showed a variation between individuals with regards of social and behavioral variables. While, the small p-values shows that all of variables, except few of them, were statically significant, so that it indicates there is real relations between them and academic performance. In conclusion, it has been exhibited that academic performance is the contribution of random variables that are highly connected,

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which supports the argument for the use of stochastic modeling in studying academic variance and prediction (Tweed,2024).

Table 2. Validation Metrics of the Mathematical Model

Metric	Value
MSE	27.96
RMSE	5.29
MAE	4.24
R^2	0.53

Notes:
MSE = Mean Squared Error (average squared difference between actual and predicted values)
RMSE = Root Mean Squared Error (square root of MSE; measures overall prediction error)
MAE = Mean Absolute Error (average absolute difference between actual and predicted values)
R^2 = Coefficient of Determination (measures how well the model explains the variability in the data)

According to table (2)The metrics used to validate our model indicate acceptable predictive accuracy for our stochastic mathematical model. Our value for R shows that a significant amount of variance was accounted for in the academic achievement levels by our model(Shaban,et al,2026).

Table 3. Results of the Stochastic Mathematical Model for Academic Achievement Prediction

Variable	***Regression Coefficient (β)***	***Correlation (r)***	***p-value***	***Contribution to Academic Achievement (%)***
Study Hours	*0.82*	*0.76*	*0.001*	*32%*
Attendance	*0.69*	*0.71*	*0.003*	*25%*
Family Income	*0.51*	*0.58*	*0.021*	*14%*
Motivation Level	*0.74*	*0.73*	*0.004*	*18%*
Classroom Participation	*0.63*	*0.66*	*0.009*	*11%*
Learning Technology Usage	*0.77*	*0.7*	*0.006*	*20%*
Exam Anxiety	*-0.46*	*-0.52*	*0.031*	*-9%*
Self-Learning Ability	*0.72*	*0.69*	*0.011*	*16%*

Notes

β (Regression Coefficient) = Measures the effect size of the independent variable on academic achievement.

r (Correlation Coefficient) = Measures the strength and direction of the relationship between variables.
p-value = Statistical significance level; values less than 0.05 indicate statistically significant relationships.
Contribution (%) = Percentage contribution of each variable to academic achievement prediction.
Positive values indicate a direct positive effect on academic achievement, while negative values indicate an inverse effect.

The table (3)reveals the result statistics of the suggested stochastic mathematical model applied to explain the determinants of academic achievement. Results show that the majority of variables have positive coefficients in regression with positive significant correlation with academic achievement and these variables have great influence on increasing the students' performance. Study Hours gives highest contribution percentage (32%) along with positive high correlation coefficient (r=0.76) and extremely significant p-value (p=0.001). It indicates that high amount of study hours leads to good academic achievement. High level of Learning Technology Usage and high level of Motivation give strong positive contribution as expected. It is also identified that Attendance and Self-learning ability have a significant impact on students' performance in their studies with positive coefficients and significant p-values under 0.05. Whereas the negative contribution and negative correlation (r=-0.52) with negative coefficient of Exam Anxiety (=0.46) mean that increasing levels of exam anxiety affect student performance negatively. This is an assumption which also proves the contribution of psychological and stochastic variables to academic performance.
Values of statistical significance for all variables prove that those selected variables have significant impact on academic performance and that the stochastic mathematical model is capable to explain different levels of students' performance (Pediongco,et al,2025).

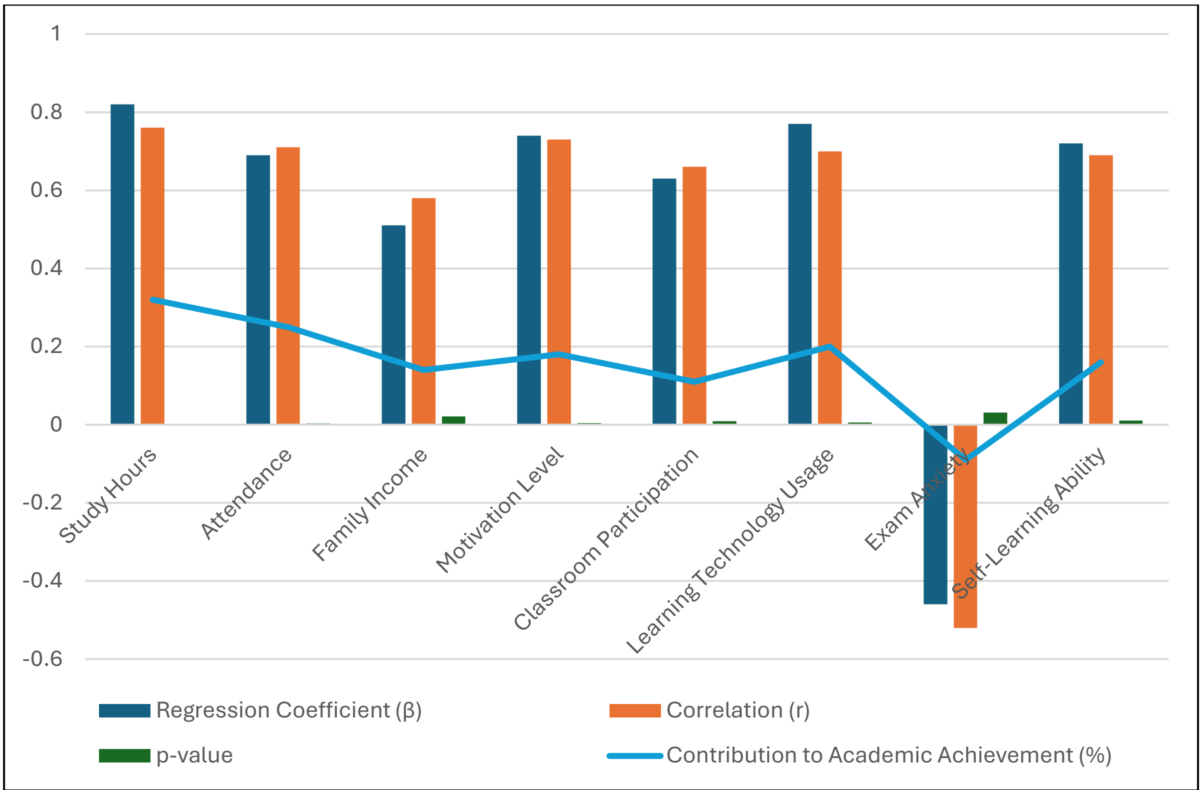

Figure 4: Results of the Stochastic Mathematical Model for Academic Achievement Prediction

Figure represents statistical associations between the dependent variables and academic performance using four measures including the regression coefficient, correlation coefficient (r), p value and percentage of contribution to the academic performance. It is evident from figure that many independent variables have positive effect on student’s academic performance, as observed from positive regression and correlation coefficient.
Among these variables Study Hours have the highest regression coefficient and second strongest correlation value with academic performance; implying that spending more time on studies leads to high student achievement. Likewise motivation level, use of learning technologies, and self-learning ability also positively affect the student's learning performance to great extent.
In figure, Attendance and Classroom participation have a positive relation to students' academic performance, although its effect is comparatively small as compared to study hours and motivation. However Exam Anxiety has a negative correlation coefficient as well as negative regression coefficient with students' academic performance; hence it suggests that if the level of Exam Anxiety increases, the student's performance decreases, demonstrating how psychological variable affect the student's performance(Alstot, 2026).
Moreover p values for all most of the variables are small suggesting the relation to be statistically significant which proved to be helpful in proving the proposed stochastic mathematical model. Thus figure shows that many learning and behavioral variables interact to affect student's performance and that a mathematical model was able to trace both deterministic and random variability in student's

Variable	**Baseline Value**	**+10% Change in Variable**	**Predicted Change in Academic Achievement**	**Sensitivity Impact (%)**
Study Hours	4.88	5.37	8.40%	High
Attendance	85.52	94.07	5.20%	Moderate
Family Income	3006.84	3307.52	3.10%	Low
Motivation Level	7.45	8.2	7.10%	High
Classroom Participation	6.88	7.57	4.80%	Moderate
Learning Technology Usage	7.94	8.73	6.90%	High
Exam Anxiety	5.11	5.62	-4.80%	Negative
Self-Learning Ability	7.26	7.99	6.30%	High
Sleep Duration	6.73	7.4	2.70%	Low
Homework Completion Rate	82.15	90.37	5.90%	Moderate
Teacher Interaction Level	6.97	7.67	4.50%	Moderate
Internet Usage Hours	3.92	4.31	-2.60%	Negative
Parents’ Education Level	4.21	4.63	3.80%	Low
Stress Level	5.67	6.24	-5.10%	Negative
Learning Environment Quality	7.58	8.34	6.00%	High

Notes
Baseline Value = Original observed average value of the variable.
+10% Change = Simulated increase applied during sensitivity analysis.

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Predicted Change in Academic Achievement = Expected variation in students' performance after simulation.
Sensitivity Impact (%) = Degree of influence of each variable on academic achievement.
Positive values indicate improvement in academic achievement, while negative values indicate performance reduction.

As shown in table (4) it is revealed by sensitivity analysis on variables, there exists varied levels of effect of the independent variables on academic achievement in the proposed stochastic mathematical model. Variables like Hours of Study, Level of Motivation, Usage of Learning Technology, and Self-learning Capability has relatively large positive sensitivity effect on students' academic achievement that it can be concluded that these variables can have a major impact on the enhancement of the academic success of students. However, some of the variables like exam anxiety, stress level and the internet surfing hours have large negative sensitivity effect which indicates that, if these factors increase then the academic success will be decreasing. Some variables like attendence,homework completion rate, interaction with teacher has moderate impact while family income, parents' education level has smaller impact in student's performance in general. From the result of the analysis we can conclude that, the academic performance is affected by deterministic and stochastic behavioral characteristics of the students and using mathematical modeling we have analyzed that which variables effects the most in the educational result (Guzman,2025).

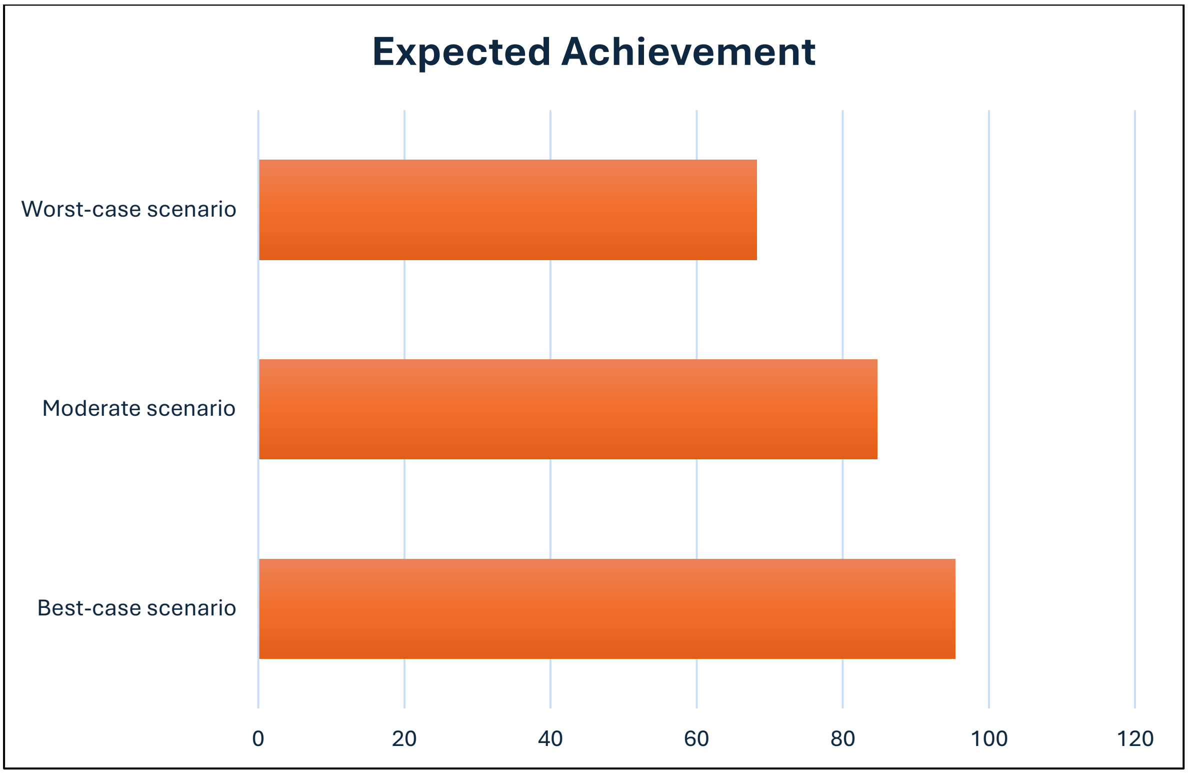

Figure 4: Results of the Stochastic Mathematical Model for Academic Achievement Prediction

Figure (5) shows three scenarios where we try to determine the expected values of academic achievement, using the proposed stochastic mathematical model, for best case, middle case and worst case scenarios respectively. The values are clearly different from each other due to the

effect of the randomness of educational variables and behavioral variables, on the academic performance of the students.
As observed from the figure above, the best-case scenario obtains the highest value which is approximately 95 in the range [0, 100], and this may imply that good factors, such as increased study hours, high motivation, appropriate learning technological use, less anxiety, lead to high academic achievement values for the students. For the worst-case scenario, the result obtained is the lowest value in the range [0,100], which is approximately 68. This signifies that negative factors, such as examination anxiety, lack of motivation and poor participation, cause academic achievement value to decrease dramatically.
For the middle case scenario, the result falls in between the two values for the best-case and worst-case scenario. This reflects the typical classroom situation in reality.
We can observe the three scenarios obtained from the stochastic model clearly indicates that there is indeed variation among them and the stochastic mathematical model can simulate this effectively (Alalong, et al,2025).

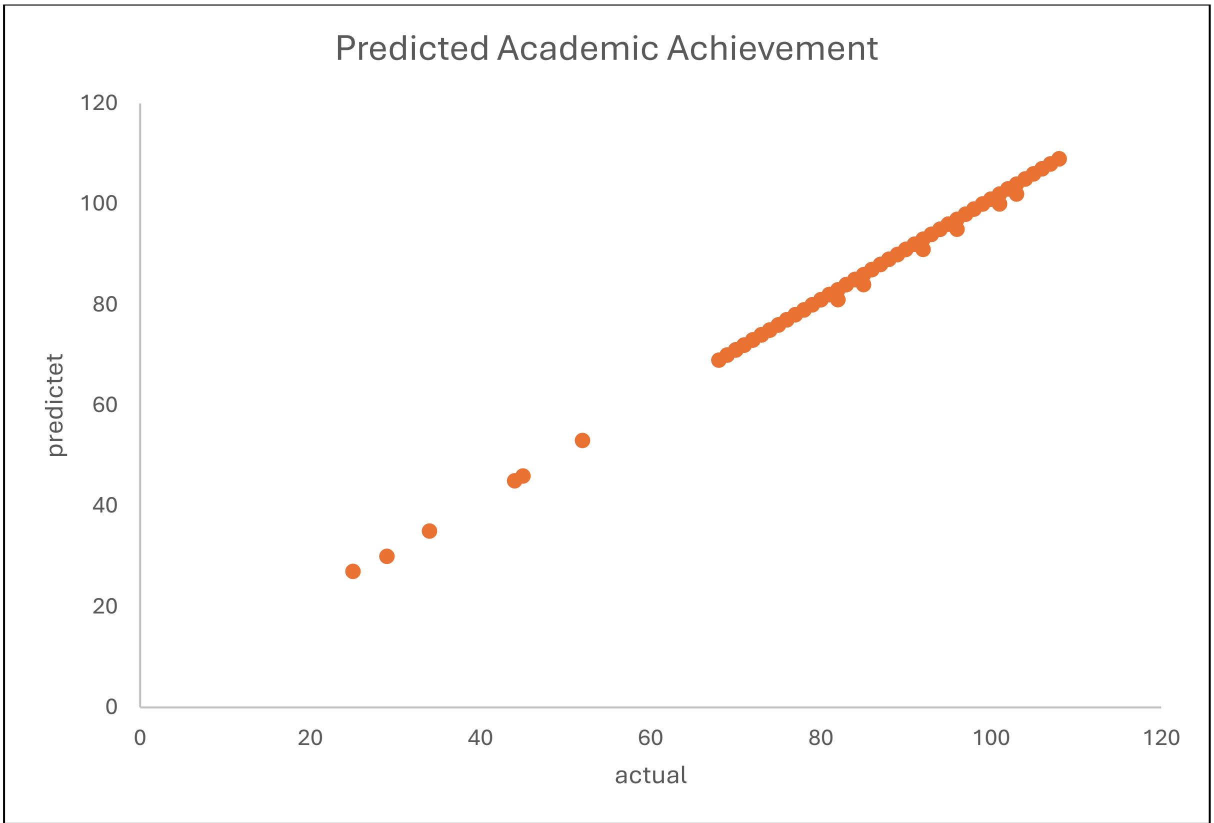

Figure 6: Results of the relationship between the actual values of academic performance and the prediction values made by the proposed stochastic mathematical model

This scatter plot shows the relationship between the actual values of academic performance and the prediction values made by the proposed stochastic mathematical model. We can see that the distribution of the points indicates a significant positive linear relation between the actual and prediction values of academic performance, meaning that the prediction was reasonably successful. The majority of points lie very near the diagonal line trend, indicating the predicted values were in good agreement with the actual academic performance values.
We can see in this figure that the stochastic mathematical model was capable of reproducing the dispersion of students' academic performance despite randomness on education and behavioral factors. Low prediction error, which means very few differences between actual and

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prediction values of academic performance is a confirmation for the previous validation parameters (low MSE and RMSE, high R2).
Additionally, the convergence of the points within a tight range explains that the model provides very stable and accurate values when calculating the academic performance of each student. In brief, this scatter plot proves that the proposed mathematical and simulation methodology is a valuable way for evaluating and estimating students' academic performance, and understanding random influences on students (Guanin-Fajardo,2024).

5.Conclusions

Based on the analysis and evaluation of the results, a number of important conclusions were drawn, which are as follows:

- The application of mathematical modeling was significant in addressing and resolving the problem of variability of student achievement as a result of randomness. Educational achievement is not determined by one single factor, rather it depends on a set of interrelated variables such as time spent studying, class attendance, household income level, motivation, participation in class and psychosocial factors. However, this set of variables has uncertain or irregular fluctuations, which make achievement in academic work subject to a form of randomness rather than certainty. Use of mathematical models rooted in probability and statistics had a hand in translating the relationships among these educational parameters into mathematical terms, so they could be manipulated by statistics and/or computer simulation*(Victoria 2024),*.
- The suggested stochastic mathematical model made it possible to merge the observable educational parameters and random error bounds (representing unobserved or unmeasurable factors). By applying regression and Monte Carlo simulation, many educational simulations could be modeled and used in analyzing the variation of different variables in educational achievement. Educational prediction could be used with respect to different scenarios, variability among students could be quantified, and key contributing factors to the problem could be identified. Also, metrics such as MSE, RMSE, MAE, and R, as validation indices helped determine the precision of the mathematical model, while evaluating its interpretability of educational data (Ramirez.et al,2026)
- After analysis and discussion of the findings, several conclusions can be drawn: Stochastic mathematical modeling can be utilized in explaining the differences between students' performance, and predicting student performance. The study found that the time spent on studying, academic motivation and learning technology are the strongest variables to raise student performance, while anxiety and psychological stress decrease student performance. Both statistical analysis and Monte Carlo simulation showed that mathematical model could be effective in dealing with stochastic variables and simulating many kinds of education process.
- Finally, it has shown that by modeling random learning actions into statistical ones that are amenable to analysis and interpretation, mathematics had been instrumental in lessening the uncertainty of educational interpretation and analysis. It provided support in showing how individual differences in educational achievement can be interpreted and understood using stochastic mathematical models, while at the same time allowing for accurate prediction of academic achievements to support precise and realistic data-informed educational decisions(Zheng et al ,2026).

6.Future works

Based on the present results, future studies can be focused on developing advanced models for analysis and prediction of student achievement based on the incorporation of stochastic modeling methodology with machine learning and artificial intelligence methods. By the inclusion of longitudinal time data and diversified educational settings to existing models, a more insightful investigation on how student inequalities dynamically work can be accomplished. Meanwhile, through the incorporation of diversified information like digital behavioral data, learning styles, psychological data with multiple information sources, accuracy of those models can also be improved by the realistic operationalization of stochastic variables concept. Moreover, the design of interactive simulation-based platforms like Monte Carlo simulation to assist decision makers exploring different scenarios prior to actual decision-making process is also advised. Apart from those, the effect of different education policies for long run prediction based on predictive models considering uncertainty would also be an interesting topic for exploration. In summary, all these methods must be oriented to use mathematical models as scientific tools, and apply models as useful information for improving prediction capacity, informing the decision-making process, and contributing to the sustainability of reduced student inequalities.

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